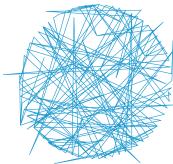


Motion problem formulation using system theory: examples

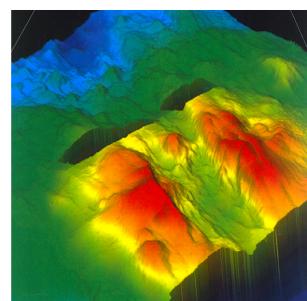
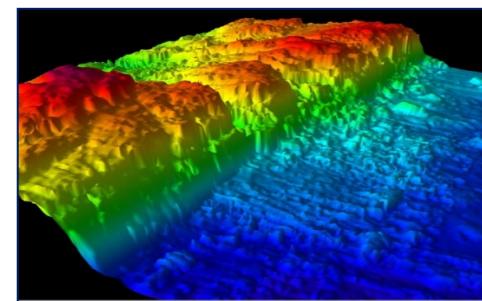
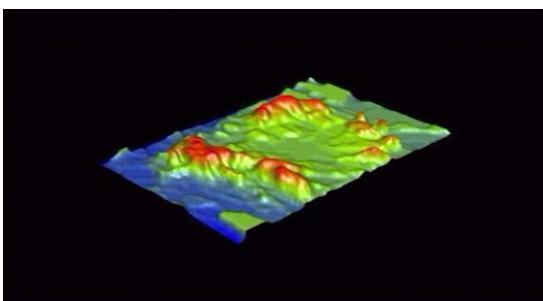
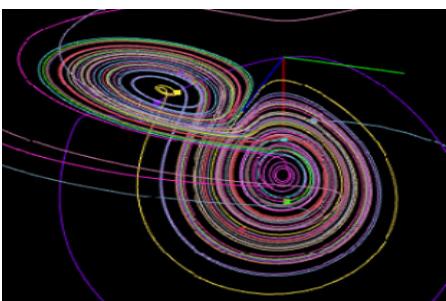
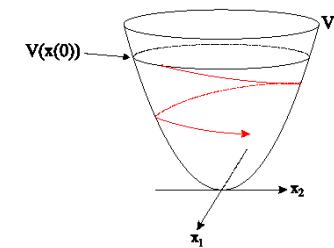
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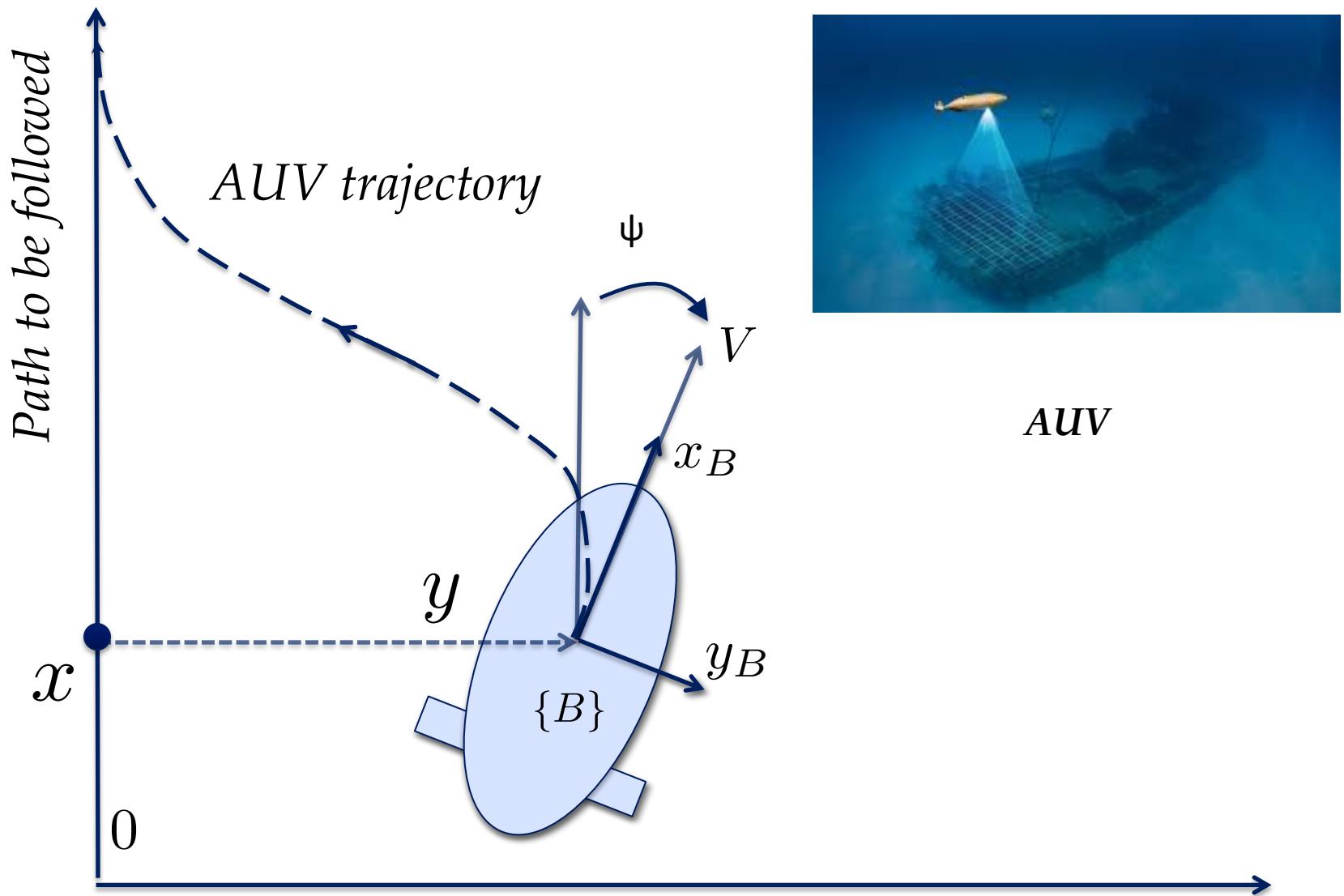


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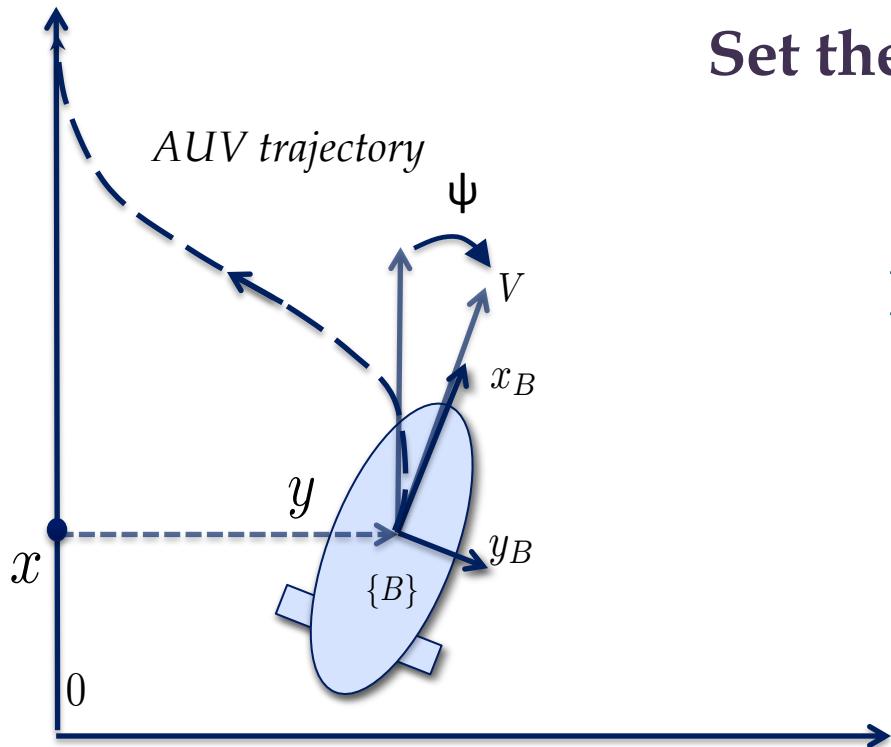
AUV Path Following

Path following formulation



AUV Path Following

Path following formulation

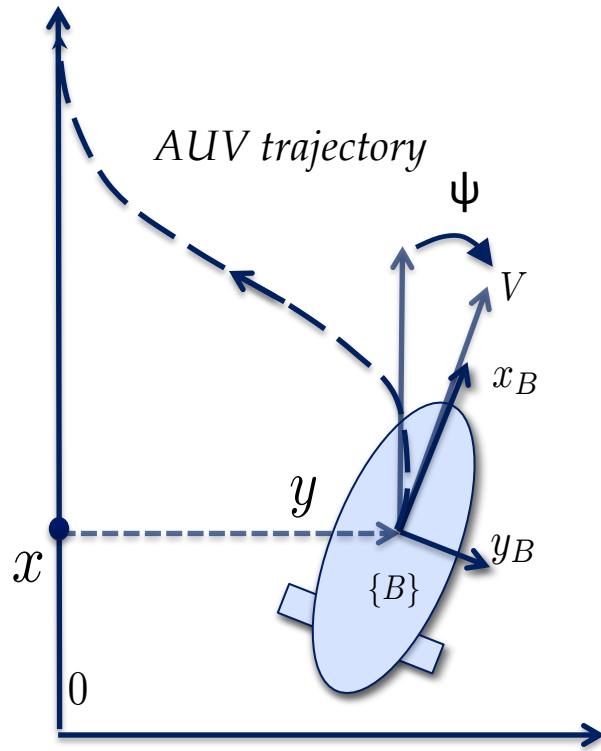


Set the speed of the AUV equal to $V > 0$ (constant)

Recruit the heading angle so that the lateral distance to the vertical path will converge to 0!

AUV Path Following

Path following formulation



Plant Model I:

$$\frac{dy(t)}{dt} = V \sin \psi(t)$$



Objective

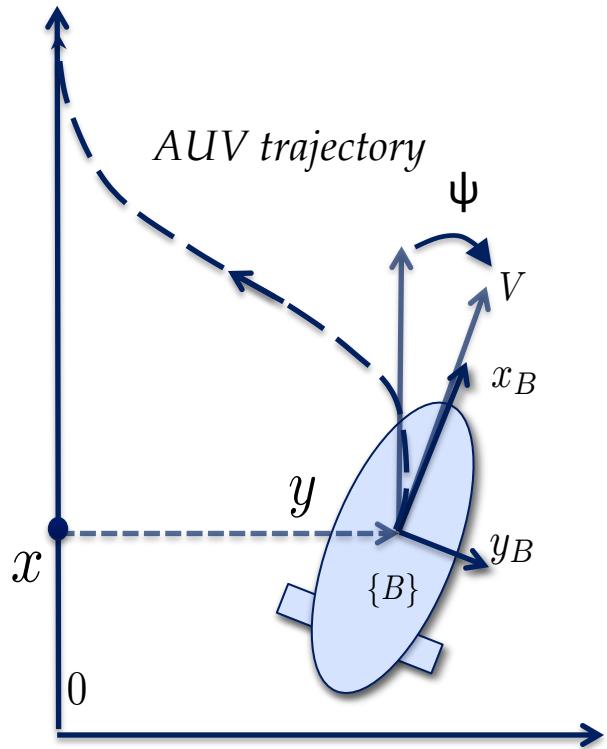
Compute $\psi(t)$ so that $\lim_{t \rightarrow \infty} y(t) = 0$

AUV Path Following

$y(t)$: deviation from the path (path following ERROR)

Objective:
reduce the error to 0!

**Simplified
(linearized) model**



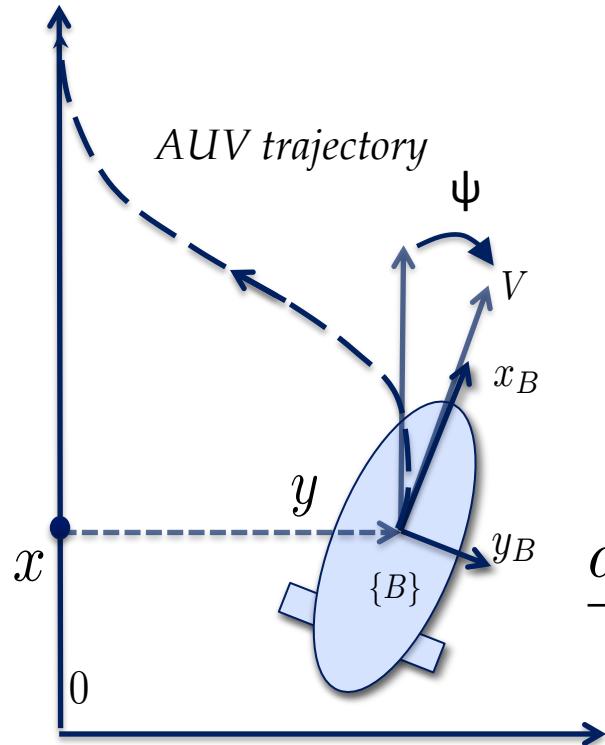
$$\frac{dy(t)}{dt} = V \sin \psi(t)$$

↓
small $\psi(t)$

$$\frac{dy(t)}{dt} = V \psi(t)$$

AUV Path Following

Objective:
reduce the error to 0!



$$\frac{dy(t)}{dt} = V\psi(t)$$

make $\psi(t) = -\frac{1}{V}y(t)$

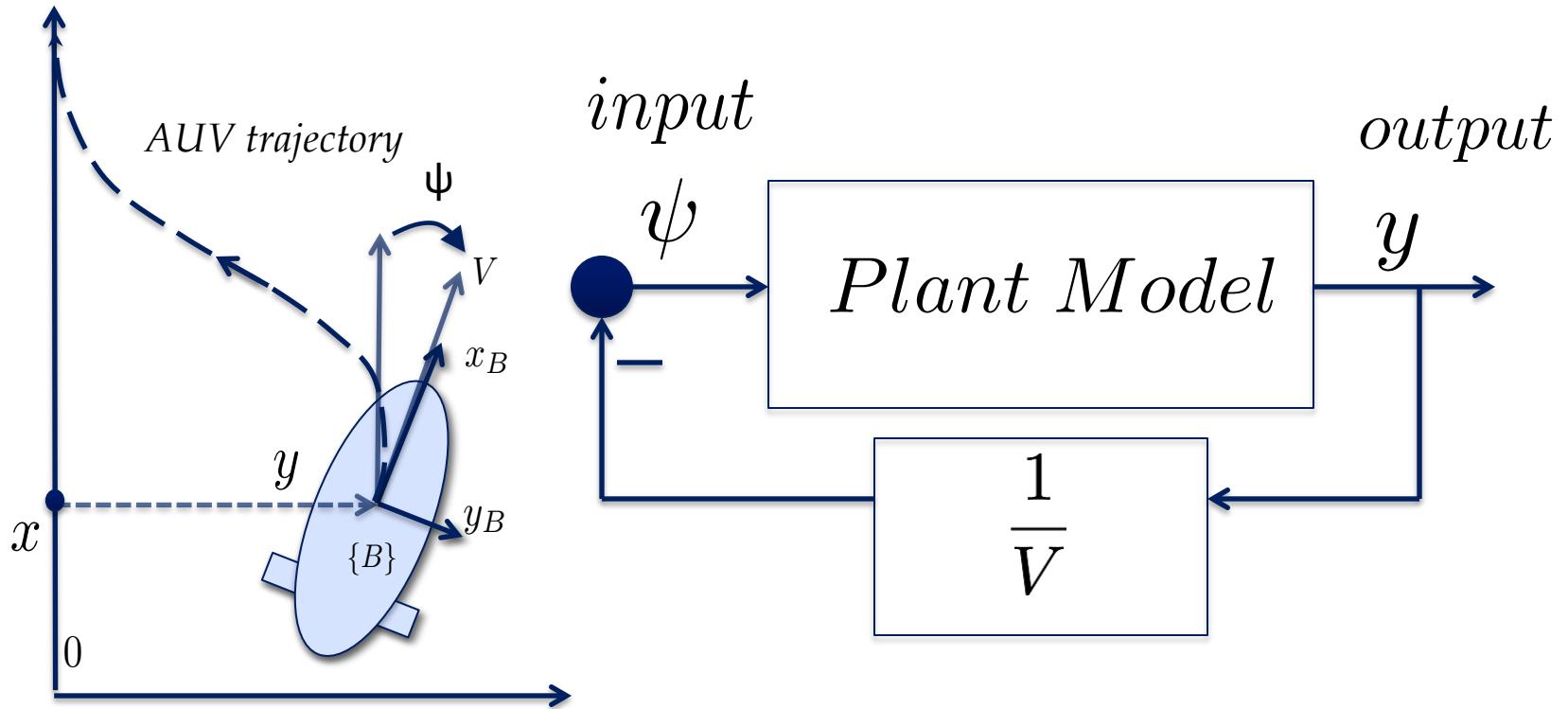
for small y

$$\frac{dy(t)}{dt} = -y(t), \text{ i.e. } \frac{dy(t)}{dt} + y(t) = 0$$

$y(t) \rightarrow 0$ (success!)

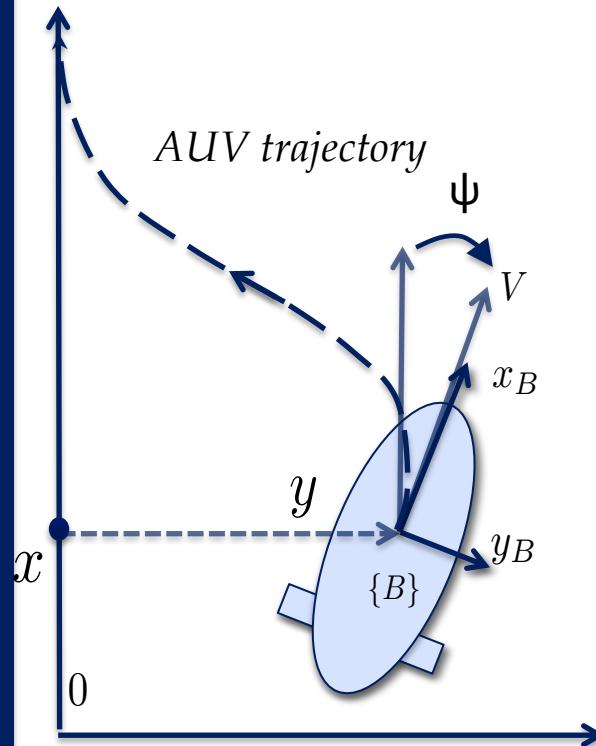
AUV Path Following

Feedback Control system implementation



AUV Path Following

Path following formulation

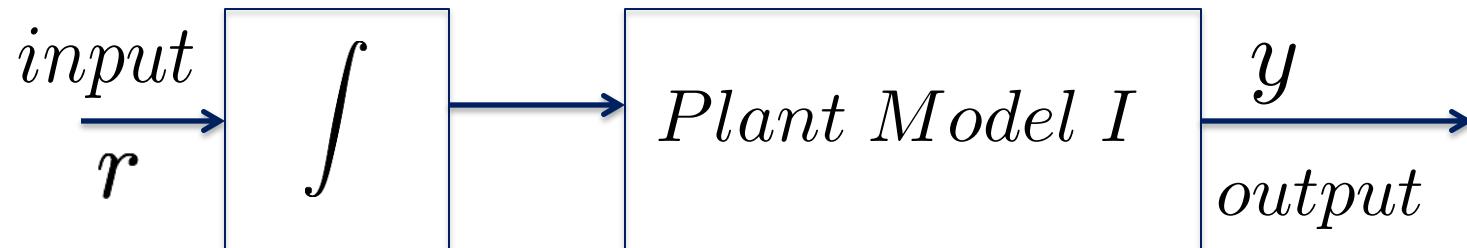


Plant Model II:

$$\frac{dy(t)}{dt} = V \sin \psi(t)$$

$$\frac{d\psi(t)}{dt} = r$$

r (yaw rate) is the new input

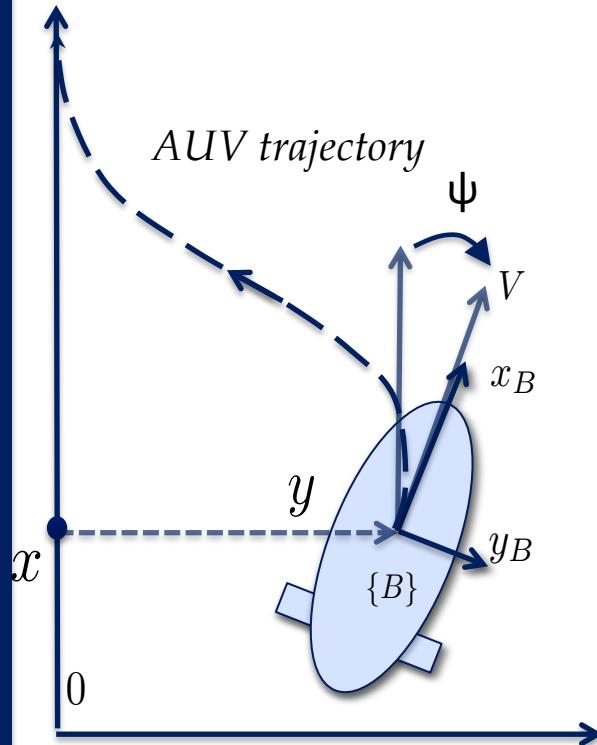


Objective:

Compute $r(t)$ so that $\lim_{t \rightarrow \infty} y(t), \psi(t) = 0$

AUV Path Following

Path following formulation



Plant Model II:
Simplified (linearized) model

$$\frac{dy(t)}{dt} = V \sin \psi(t)$$

$$\frac{d\psi(t)}{dt} = r$$

small $\psi(t), y(t)$

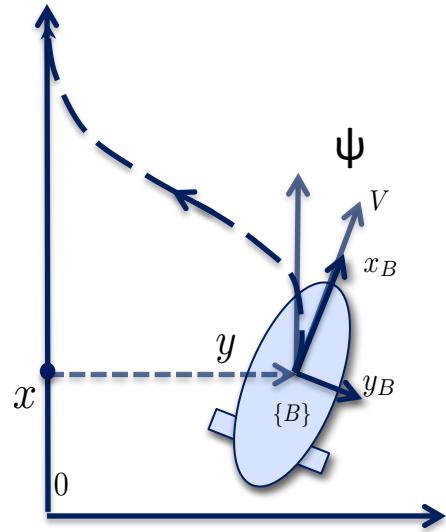


$$\frac{dy(t)}{dt} = V\psi(t); \quad \frac{d\psi(t)}{dt} = r$$

$$\frac{d^2y(t)}{dt^2} = Vr(t)$$

AUV Path Following

Path following formulation



Plant Model II:
Simplified (linearized) model

$$\frac{d^2y(t)}{dt^2} = Vr(t)$$

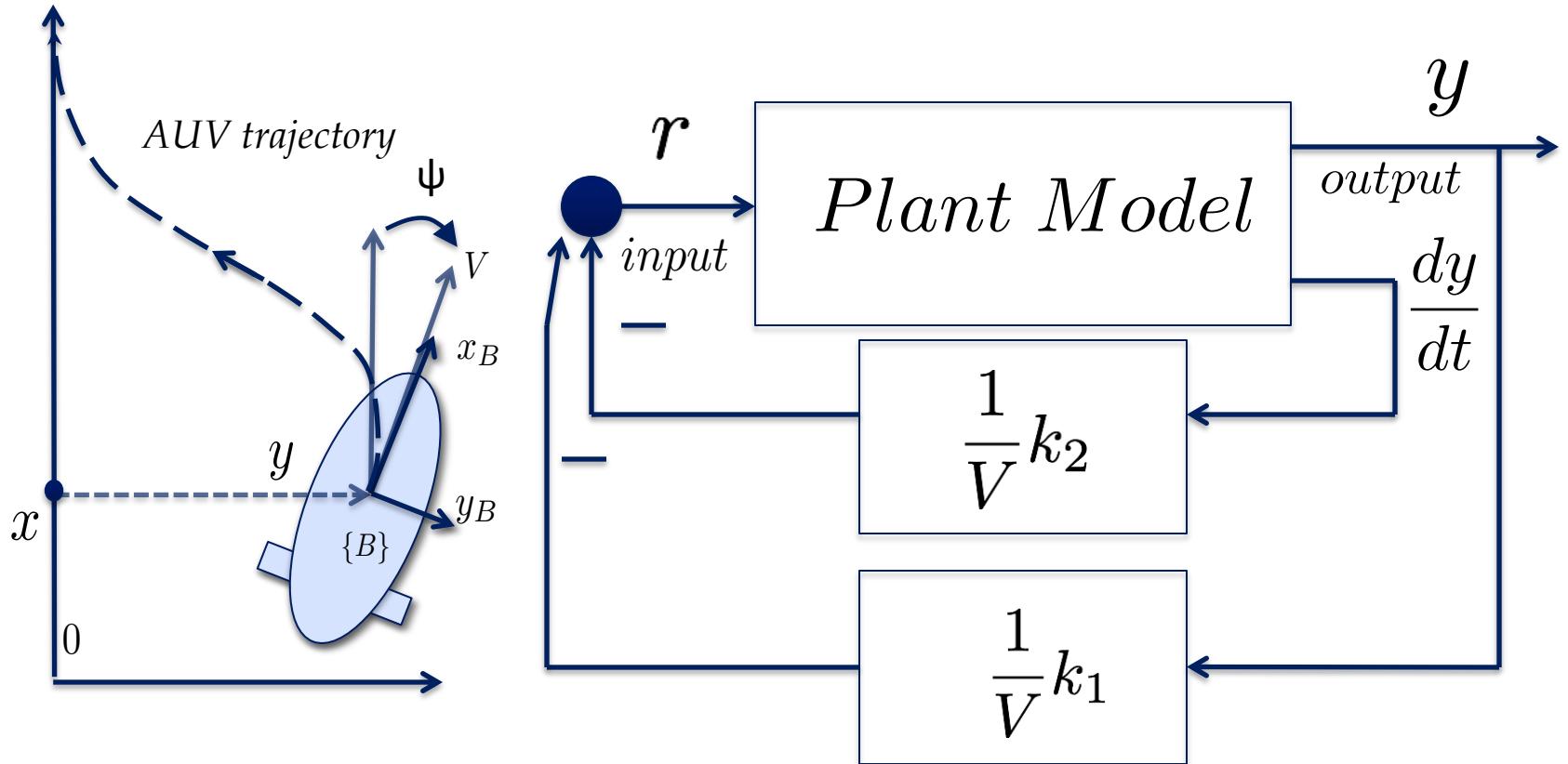
$$\text{make } r(t) = -\frac{1}{V}(k_1y(t) + k_2\frac{dy(t)}{dt})$$

$$\frac{d^2y(t)}{dt^2} + k_2\frac{dy(t)}{dt} + k_1y(t) = 0$$

$$y(t), \frac{dy(t)}{dt} \rightarrow 0 \text{ (success!)}$$

AUV Path Following

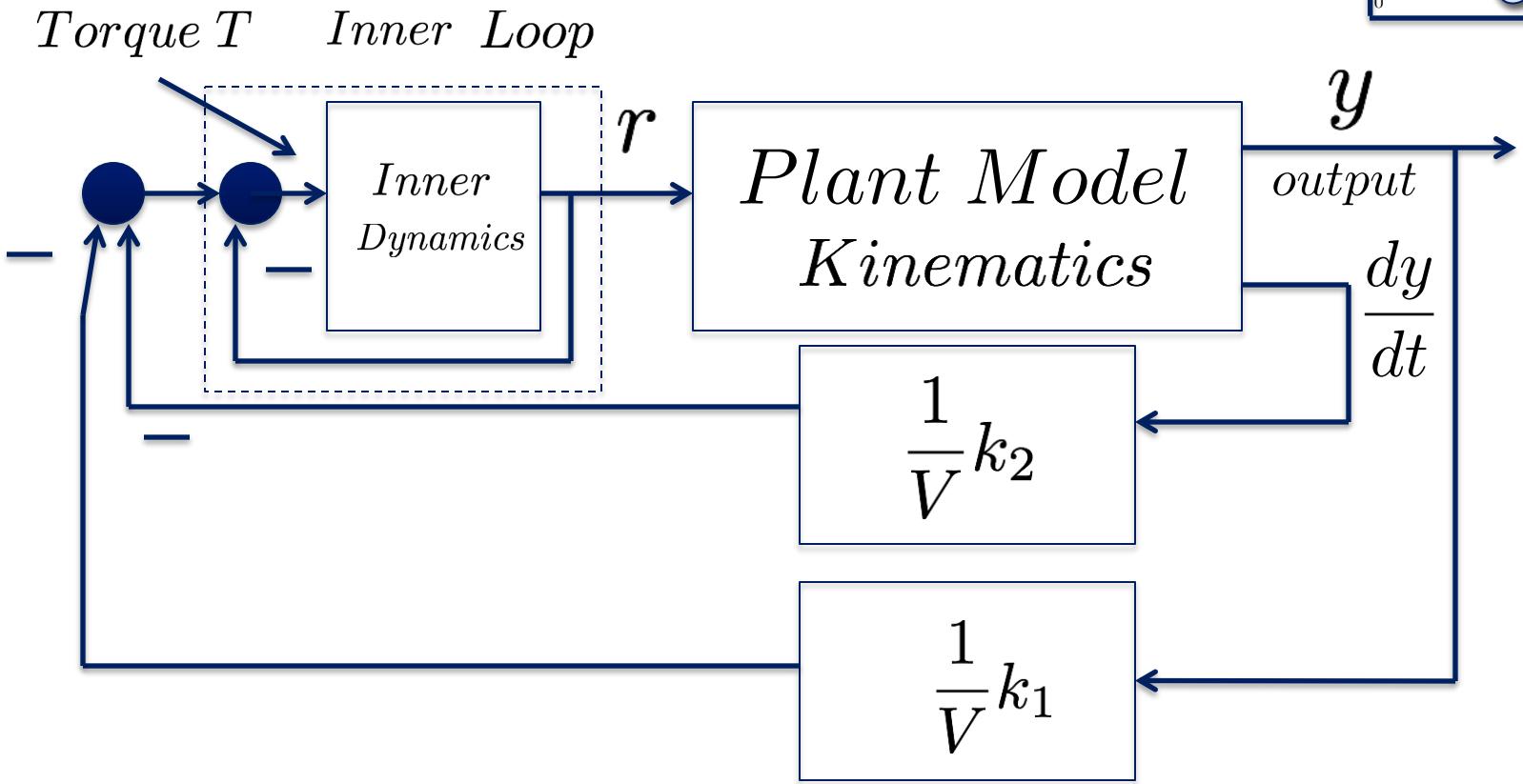
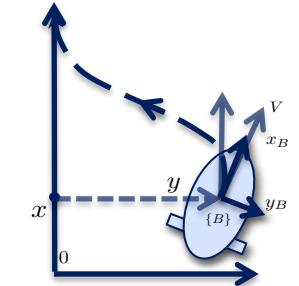
Feedback Control system implementation
("outer-loop")



Feedback from lateral error and the corresponding rate!

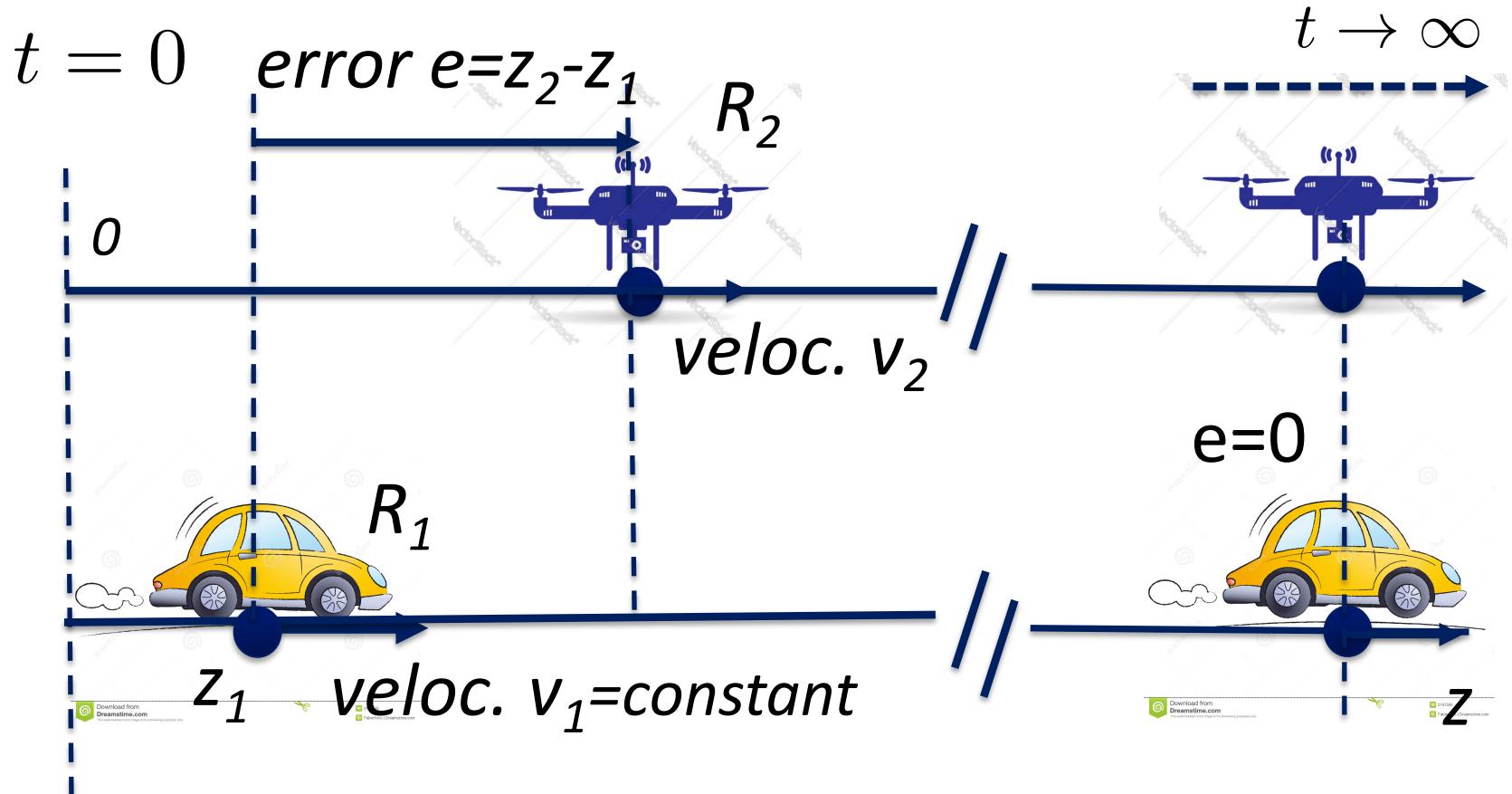
AUV Path Following

Feeback Control system implementation
("inner" and "outer-loop")



Feeback from lateral error and the corresponding rate!

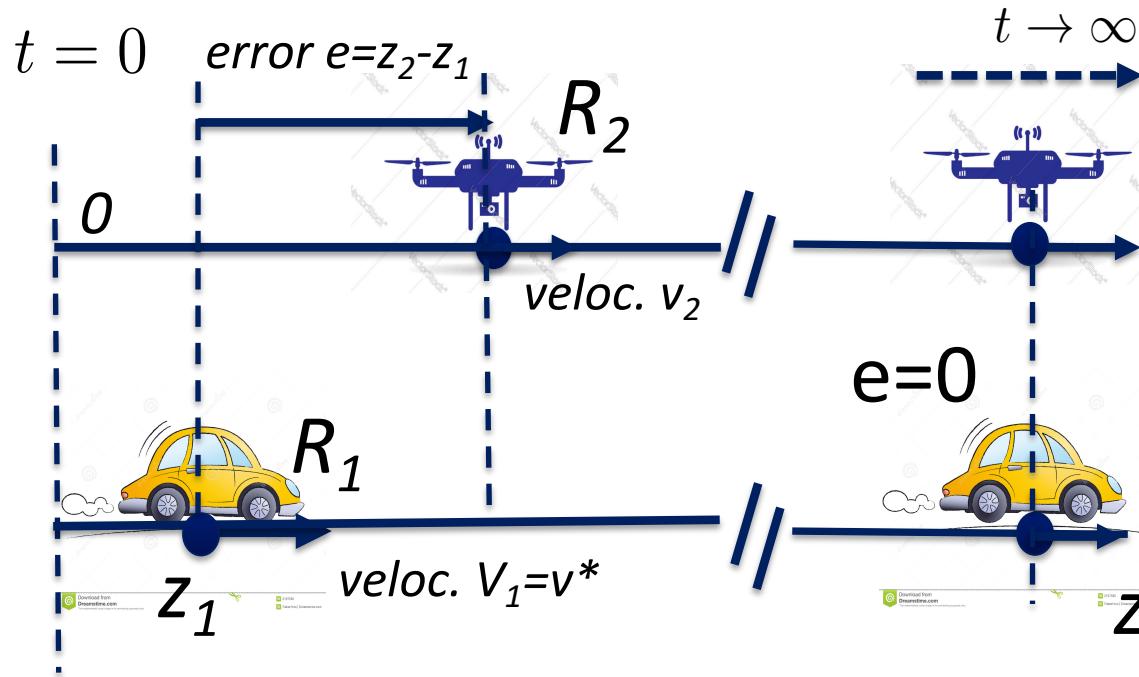
Cooperative Motion Control (land and aerial robots)



Robot 1 travels at known constant speed $v_1=v^$*

Robot 2 maneuvers to “stay on top” Robot 1

Cooperative Motion Control (land and aerial robots)



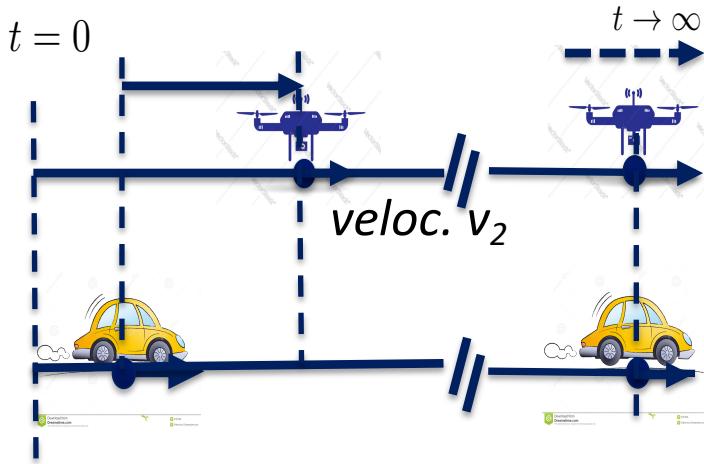
Objective:

Determine $v_2(t)$ so that

$$\lim_{t \rightarrow \infty} e(t) = z_2(t) - z_1(t) = 0$$

Reduce the error to 0!

Cooperative Motion Control (land and aerial robots)



$$e(t) = z_2(t) - z_1(t)$$



Plant Model I:

$$\frac{de(t)}{dt} = \frac{dz_2(t)}{dt} - \frac{dz_1(t)}{dt}$$

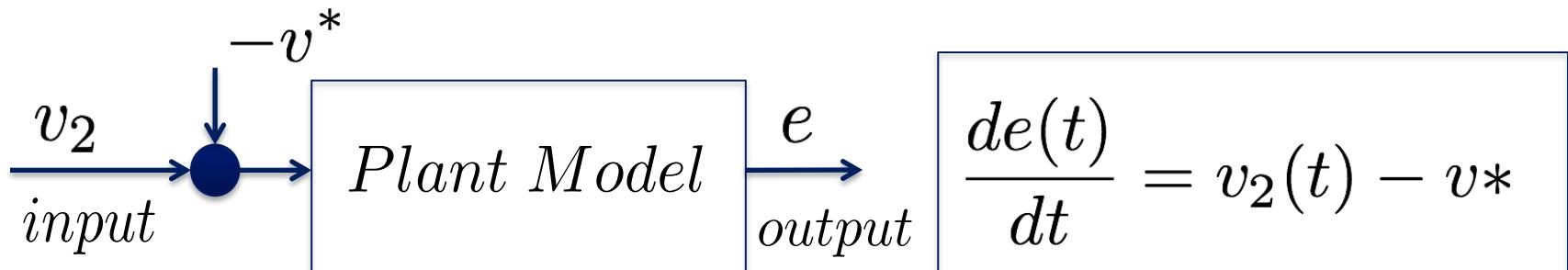
↓

$$\frac{de(t)}{dt} = v_2(t) - v^*$$

Determine $v_2(t)$ so that

$$\lim_{t \rightarrow \infty} e(t) = 0$$

Cooperative Motion Control (land and aerial robots)



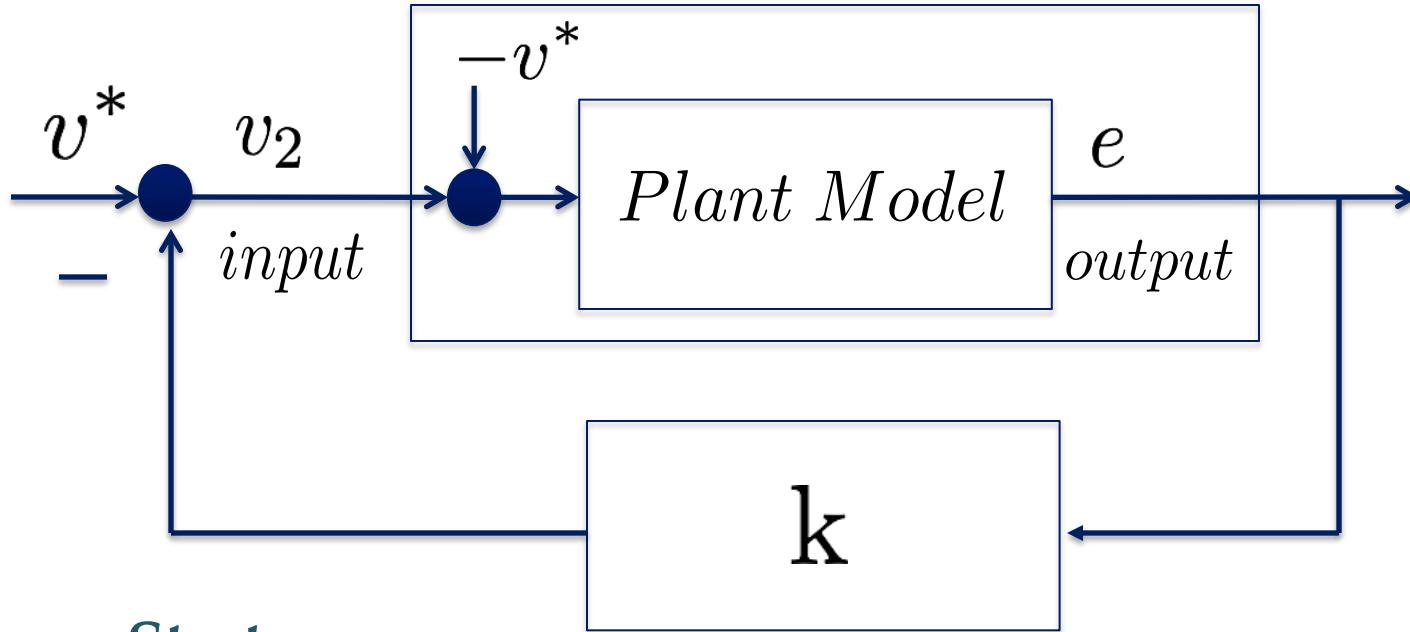
$$\text{make } v_2(t) = v^* - k(e(t))$$

$$\frac{de(t)}{dt} = -ke(t)$$

$$\lim_{t \rightarrow \infty} e(t) = 0$$

Cooperative Motion Control (land and aerial robots)

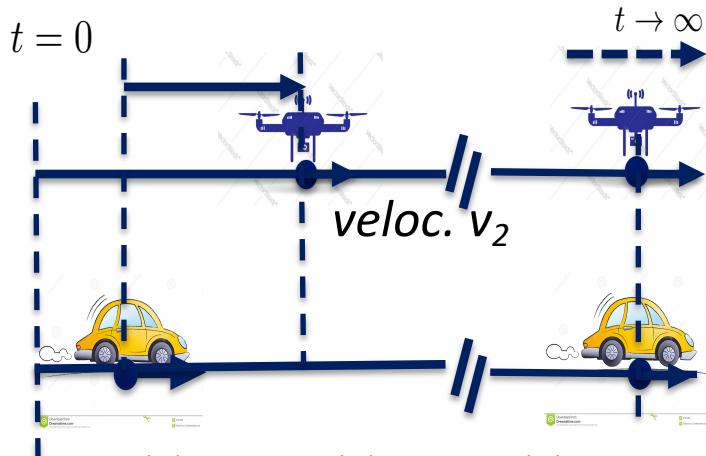
Feedback Control system implementation



Strategy:

1. Decrease the speed when R2 is ahead of R1
2. Increase the speed when R2 is behind R1
3. K sets the “rate of coordination”

Cooperative Motion Control (land and aerial robots)



Plant Model II:
(force as an input)

Kinematics

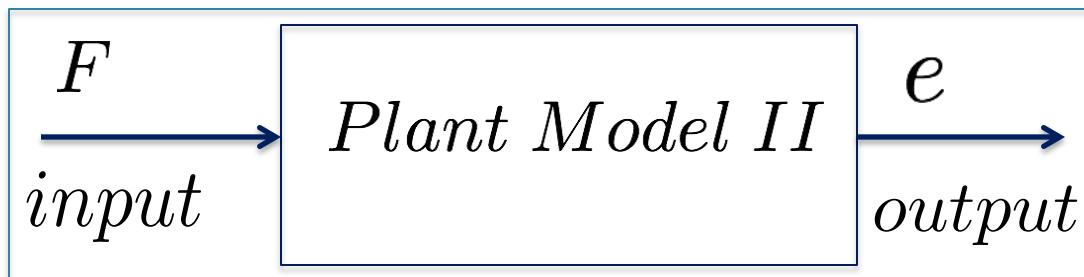
$$\frac{de(t)}{dt} = v_2(t) - v^*$$

Dynamics

$$F = m \frac{dv_2(t)}{dt}; \quad m = 1 \text{ kg}$$

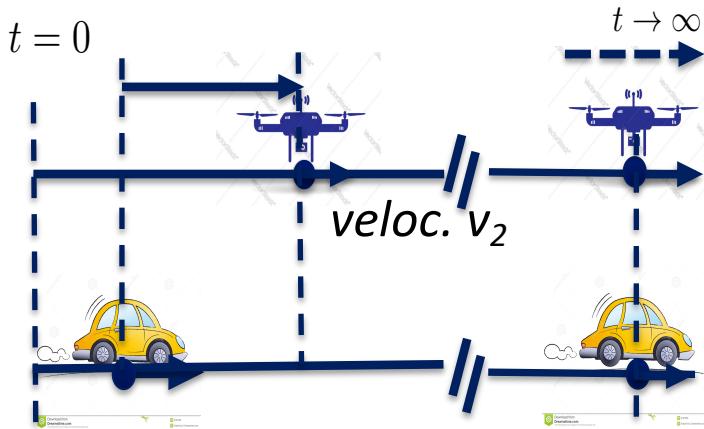
$$F = \frac{d^2 e(t)}{dt^2}$$

NO DRAG!



Determine $F(t)$ so that $\lim_{t \rightarrow \infty} e(t) = 0$

Cooperative Motion Control (land and aerial robots)



$$e(t) = z_2(t) - z_1(t)$$

Plant Model II:
(force as an input)

$$F = \frac{d^2 e(t)}{dt^2}$$

$$\text{make } F(t) = -(k_1 e(t) + k_2 \frac{de(t)}{dt})$$



$$\frac{d^2 e(t)}{dt^2} + k_2 \frac{dy(t)}{dt} + k_1 e(t) = 0$$

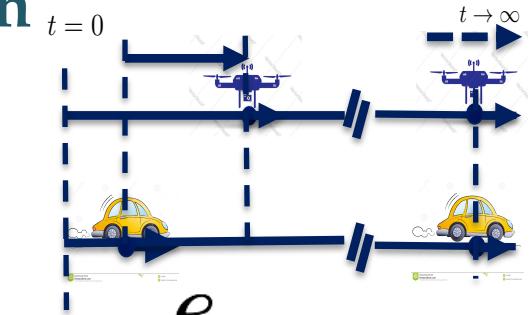
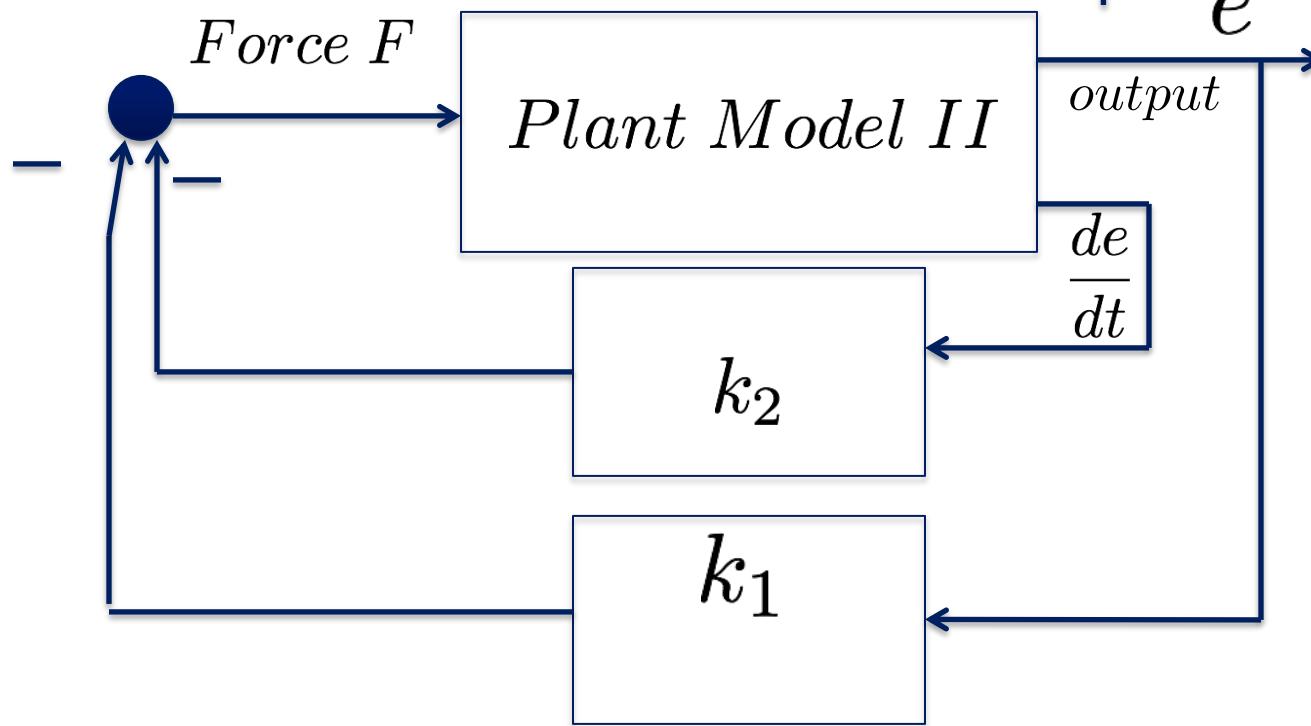


$$e(t), \frac{de(t)}{dt} \rightarrow 0 \text{ (success!)}$$

No need to know v^* !

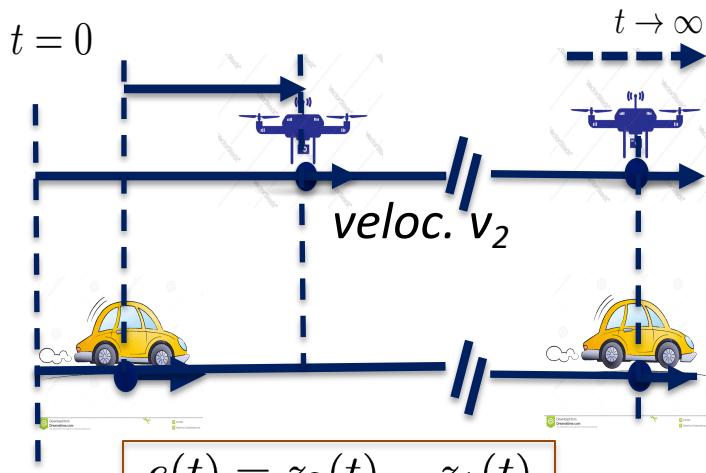
Cooperative Motion Control (land and aerial robots)

Feeback Control system implementation ("inner" and "outer-loop")



Feeback coordination error and the corresponding rate!

Cooperative Motion Control (land and aerial robots)



$$\begin{aligned} e(t) &= z_2(t) - z_1(t) \\ \frac{de(t)}{dt} &= v_2(t) - v^* \\ \frac{d^2e(t)}{dt^2} &= \frac{d^2v_2(t)}{dt^2} \end{aligned}$$

Plant Model III: (system with DRAG)

Kinematics $\frac{de(t)}{dt} = v_2(t) - v^*$

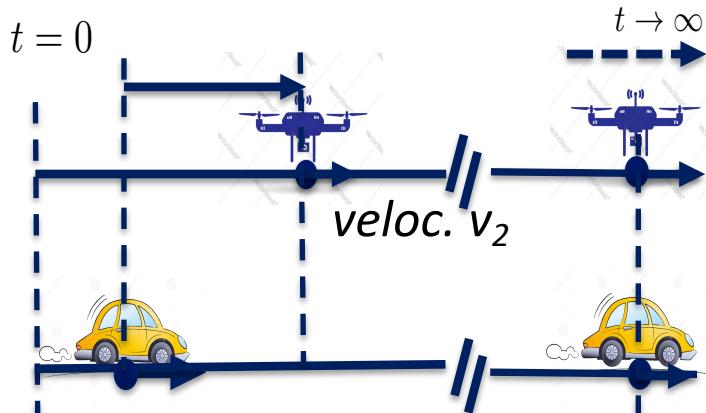
Dynamics $F - \beta v_2^2 = \frac{d^2v_2(t)}{dt^2}$

$$F = \beta(v^* + \frac{de(t)}{dt})^2 + \frac{d^2e(t)}{dt^2}$$



Determine $F(t)$ so that $\lim_{t \rightarrow \infty} e(t) = 0$

Cooperative Motion Control (land and aerial robots)



**Linearization technique
(from nonlinear to linear app.)**

$$F = \beta(v^* + \frac{de(t)}{dt})^2 + \frac{d^2e(t)}{dt^2}$$

1. Compute equilibrium (trimming) condition corr. to

$$e(t) = 0; \frac{de(t)}{dt} = 0 \longrightarrow F^* = \beta(v^*)^2$$

2. Perturb about equilibrium conditions

$$F^* + \delta F = \beta(v^*)^2 + 2\beta v^* \frac{de(t)}{dt} + \beta \left(\frac{de(t)}{dt} \right)^2 + H.O.T + \frac{d^2e(t)}{dt^2}$$

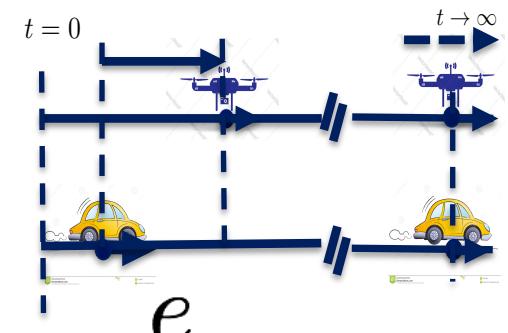
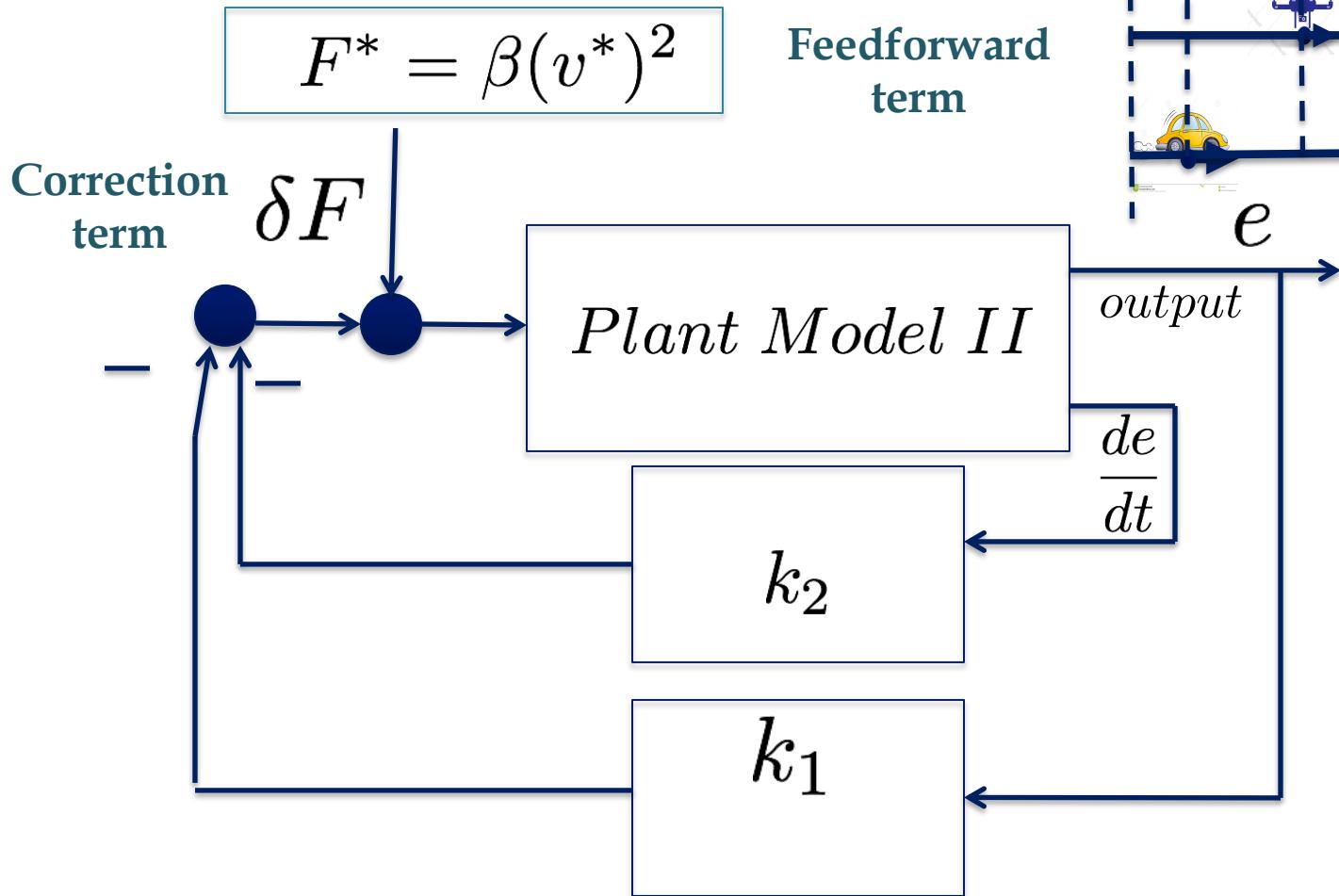
equilibrium condition

neglect higher order terms

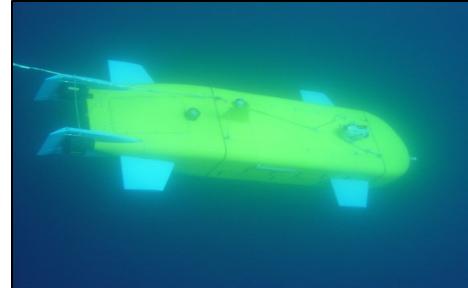
$$\frac{d^2e(t)}{dt^2} + 2\beta v^* \frac{de(t)}{dt} = \delta F \quad \text{Linear}$$

Cooperative Motion Control (land and aerial robots)

Feeback Control using linearization

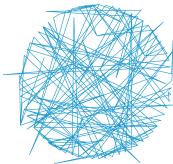


Feeback coordination error and the corresponding rate!



Motion problem formulation using system theory: examples

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